THE PLACEMENT TEST FOR PRECALCULUS/CALCULUS I - Fall, 2001

Please read the following carefully:

Your score on this test (in Part B), your math SAT or ACT score, and the information you are asked to provide in Part A, will be used to evaluate your mathematical knowledge and abilities and to place you in one of three groups:

1. those who can start Calculus I right away;
2. those who have some weaknesses in either algebra and/or trigonometry that can be remedied by taking a precalculus course or a year-long precalculus/calculus course; and
3. those whose background in mathematics is too weak and to whom we would recommend a major which does not require Calculus I and II.

Part A. BASIC INFORMATION: (Please print)

1. Name: ____________________________________________

   SSN: ___________________ Major: ______________________

2. Please mark all courses you have taken in high school or college:

   _____ Algebra I          _____ Algebra II
   _____ College Algebra    _____ College Trigonometry
   _____ Trigonometry       _____ College Algebra and Trigonometry
   _____ Precalculus        _____ Calculus
   _____ Algebra and Applications _____ Applied Algebra
   _____ AP Calculus        _____ Others (specify): ____________

3. When (senior year, this past summer, ...) and where (high school, community college, ...) did you take your last course in mathematics? What was it?

4. My personal preference would be to take (please circle one):

   _____ Precalculus     _____ Year-long Precalculus/Calculus
   _____ Calculus I      _____ Math for non-science majors


_____________________________________________________

TEST SCORES: (Do not fill this section.)

Basic Algebra: _____  Intermediate Algebra: _____  Total Algebra Score: _____

Trigonometry Score: _____  Total Placement Test Score: _____

SAT or ACT Score: _____  Recommended Placement: ____________
Part B: THE PRECALCULUS/CALCULUS I PLACEMENT TEST:
The test consists of three parts:
I. Basic Algebra,
II. Intermediate Algebra and
III. Trigonometry.
Please carefully read the following instructions before starting the test.

INSTRUCTIONS:
(1) For each question provide only one answer.
(2) Do not use a calculator.
(3) NA stands for none of the above.
(4) For all graphs, the horizontal axis is the $x$ –axis and the vertical axis is the $y$ –axis.

I. BASIC ALGEBRA:
1. $\frac{5}{4} - \frac{3}{2} + \frac{1}{3}$
   \begin{align*}
   (a) \frac{3}{5} & \quad (b) \frac{1}{12} \quad (c) \frac{37}{12} \quad (d) -\frac{15}{24} \quad (e) \text{NA}
   
   \end{align*}
   
2. $(-2)^3$ is equal to
   \begin{align*}
   (a) -8 & \quad (b) 8 \quad (c) \frac{1}{8} \quad (d) -\frac{1}{8} \quad (e) \text{NA}
   
   \end{align*}
   
3. Simplify $5x + 3(x - y) + y$.
   \begin{align*}
   (a) 4(2x - y) & \quad (b) 2(4x - 3y) \quad (c) 2(4x - y) \quad (d) 8x - y \quad (e) \text{NA}
   
   \end{align*}
   
4. Let $L$ be a constant. The solution $x$ of the equation $2x + 7 = Lx - 4$.
   \begin{align*}
   (a) \frac{4}{L - 2} & \quad (b) \frac{2x - 11}{L} \quad (c) \frac{Lx - 11}{2} \quad (d) \frac{11}{L - 2} \quad (e) \text{NA}
   
   \end{align*}
   
5. If the enrollment of a college triples every 10 years, then by what factor does it increase over a 30 year period?
   \begin{align*}
   (a) 3 & \quad (b) 6 \quad (c) 9 \quad (d) 27 \quad (e) 30
   
   \end{align*}
   
6. Which of the following graphs is the graph of $x + y = 1$? The horizontal axis is the $x$ –axis and the vertical axis is the $y$ –axis.
   \begin{align*}
   (a) & \quad (b) & \quad (c) & \quad (d)
   
   \end{align*}
   
7. The equation of the line with slope $-3$ and $y$-intercept $(0,2)$ is
   \begin{align*}
   (a) y = 3x + 2 & \quad (b) y = -3x - 2 \quad (c) y = 3x - 2 \quad (d) y = -3x + 2 \quad (e) y = 2x - 3
   
   \end{align*}
8. One of the solutions of the equation: \(2x^2 + 3x - 2 = 0\) is
\[
(a) \ -1 \quad (b) \ 2 \quad (c) \ 1 \quad (d) \ -\frac{1}{2} \quad (e) \ \frac{1}{2}
\]

9. The function \(f(x) = \frac{x + 2}{(2x + 1)(x - 3)}\) is defined at all real numbers except
\[
(a) \ -2 \quad (b) \ -\frac{1}{2} \quad (c) \ 3 \quad (d) \ -\frac{1}{2}, 3 \quad (e) \ -2, -\frac{1}{2}, 3
\]

10. The graph of \(y = \frac{3x + 2}{x^2 - 1}\) crosses the \(x\)-axis at \(x = \)
\[
(a) \ -\frac{2}{3} \quad (b) \ \frac{3}{2} \quad (c) \ 1 \quad (d) \ -1 \quad (e) \ \frac{2}{3}
\]

11. If \(f(x) = x^2\) and \(g(x) = \sqrt{2 + x}\), then \(f(-2) + g(2) = \)
\[
(a) \ -4 \quad (b) \ 2 \quad (c) \ -2 \quad (d) \ -6 \quad (e) \ 4 \quad (f) \ 6
\]

12. Which of the following graphs best resembles the graph of \(y = -(x + 2)^2 + 3\)?

\[
(a) \quad (b) \quad (c) \quad (d)
\]

13. The width of a rectangular garden is one third of its length. If the total perimeter of the garden is 48 feet, then the **width** of the garden is:
\[
(a) \ 18 \text{ feet} \quad (b) \ 6 \text{ feet} \quad (c) \ 12 \text{ feet} \quad (d) \ 10.5 \text{ feet} \quad (e) \ \text{NA}
\]

14. If the total perimeter of a circle is \(6\pi\), then the area of the circle is:
\[
(a) \ 9 \quad (b) \ 36 \quad (c) \ 9\pi \quad (d) \ 36\pi \quad (e) \ \text{NA}
\]

II. **INTERMEDIATE ALGEBRA**:

1. Which of the following: (i) \(x - 2\), (ii) \(x + 2\), (iii) \(x^2 + 4\), are factors of \(x^4 - 16\)?
\[
(a) \ (i) \text{ only} \quad (b) \ (ii) \text{ only} \quad (c) \ (iii) \text{ only} \quad (d) \ (i) \text{ and (ii) only} \quad (e) \ (i), (ii) \text{ and (iii)}
\]

2. If \(\frac{2}{x - 1} = 5\), then \(x\) is equal to
\[
(a) \ -\frac{3}{5} \quad (b) \ \frac{3}{5} \quad (c) \ \frac{7}{5} \quad (d) \ -\frac{7}{5} \quad (e) \ \text{NA}
\]

3. Let \(a\) and \(b\) be nonzero real numbers. Then \(\sqrt{a^2 - b^2} = \)
\[
(a) \ a - b \quad (b) \ \pm (a - b) \quad (c) \ a \pm b \quad (d) \ \pm (a \pm b) \quad (e) \ \text{NA}
\]

4. The least common denominator of \(\frac{3}{x^2 + 2x} + \frac{1}{x^2 - 4}\) is
(a) $x^2 - 4$  
(b) $x^2 + 2x$  
(c) $(x^2 + 2x)(x^2 - 4)$  
(d) $x(x^2 - 4)$  
(e) NA

5. $\left( \frac{2}{5} \right)^{-2}$ is equal to

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<tbody>
<tr>
<td>(a)</td>
<td>$-\frac{4}{25}$</td>
<td>(b)</td>
<td>$-\frac{4}{5}$</td>
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6. $(9^{\frac{1}{2}})(16^{\frac{1}{4}})$

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<td>(a)</td>
<td>$6$</td>
<td>(b)</td>
<td>$12$</td>
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7. $\sqrt[3]{50x^8y^{12}}$

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<td>(a)</td>
<td>$25x^4y^6$</td>
<td>(b)</td>
<td>$25x^4y^{12}$</td>
<td>(c)</td>
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8. Simplify $\left( \frac{x^2 - 9}{3x} \right) \left( \frac{12}{2x + 6} \right)$

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<tr>
<td>(a)</td>
<td>$\frac{2(x - 3)}{x}$</td>
<td>(b)</td>
<td>$\frac{2(x + 3)}{x}$</td>
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9. If $f(x) = x^2 + 3$, then $f(x - h) =$

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<td>(a)</td>
<td>$x^2 - h^2 + 3$</td>
<td>(b)</td>
<td>$(x - h)^2 + 3$</td>
<td>(c)</td>
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10. The graph of $f(x) = \frac{1}{2}x^2 + x + 1$ is given below. Find the area of the rectangle.

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<tr>
<td>(a)</td>
<td>$2$</td>
<td>(b)</td>
<td>$2.5$</td>
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11. The graph of $f(x)$ is given below. Which one of the following values is closest to $x$ if $f(x) = 4$?

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<td>(a)</td>
<td>$0$</td>
<td>(b)</td>
<td>$-1$</td>
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12. Which of the following curves best resembles the graph of $f(x) = e^x$?

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III. TRIGONOMETRY:

1. The two legs of a right triangle given below have lengths 3 and 5, respectively. Let \(A\) be the smallest angle of this right triangle. Then \(\sin(A)\) is

\[ (a) \frac{3}{5} \quad (b) \frac{4}{5} \quad (c) \frac{3\sqrt{34}}{34} \quad (d) \frac{4\sqrt{34}}{34} \quad (e) \frac{5\sqrt{34}}{34} \]

2. \([\sin(30^\circ)]^2 = \)

\[ (a) \frac{1}{4} \quad (b) \frac{3}{4} \quad (c) 1 \quad (d) \frac{1}{2} \quad (e) \frac{\sqrt{3}}{2} \]

3. If \(\sin(\theta) = a\) and \(\cos(\theta) = b\), then \(\tan(\theta)\) is equal to

\[ (a) \frac{b}{a} \quad (b) \frac{a}{b} \quad (c) \frac{a}{\sqrt{a^2 + b^2}} \quad (d) \frac{b}{\sqrt{a^2 + b^2}} \quad (e) \frac{-a}{\sqrt{a^2 - b^2}} \quad (f) \frac{b}{\sqrt{a^2 - b^2}} \]

4. \(1 - \cos^2(\theta) = \)

\[ (a) \sin(\theta) \quad (b) -\sin^2(\theta) \quad (c) \sin^2(\theta) \quad (d) \sec^2(\theta) \quad (e) -\cos(2\theta) \]

5. The graph below best represents which function?

\[ (a) \sin(2x) \quad (b) 2\sin(2x) \quad (c) \cos\left(\frac{x}{2}\right) \quad (d) \frac{1}{2}\cos(x) \quad (e) 2\cos\left(\frac{x}{2}\right) \]

6. If \(f(x) = \sin(6x)\), then \(f\left(\frac{\pi}{12}\right) = \)

\[ (a) \frac{\sqrt{3}}{2} \quad (b) \frac{\sqrt{2}}{2} \quad (c) 1 \quad (d) \frac{1}{2} \quad (e) 0 \]

7. The function \(f(x) = \tan(x)\) is not defined at \(x = \)

\[ (a) \frac{\pi}{6} \quad (b) \frac{\pi}{3} \quad (c) \frac{\pi}{4} \quad (d) \frac{\pi}{2} \quad (e) \pi \]

8. A right triangle is given below where the angle \(A = 45^\circ\). The value of \(x\) is:

\[ (a) \frac{3}{2} \quad (b) 3\sqrt{2} \quad (c) \frac{3\sqrt{2}}{2} \quad (d) \frac{3\sqrt{3}}{2} \quad (e) \text{NA} \]
4. Answers:

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<tr>
<td>1.</td>
<td>(b)</td>
<td>(e)</td>
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<td>2.</td>
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<td>3.</td>
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<td>4.</td>
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<td>5.</td>
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<td>6.</td>
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