Force Problems

This handout gives some suggestions for solving problems using Newton's Second Law. I'll assume the problem involves one or more forces acting on a single object in one or two dimensions.

1) Creating a Force Diagram

A typical problem will describe a physical situation with information about the forces and their directions. A useful first step is to create a "force diagram" (or "free-body diagram") to help organize this information.

a) Set up your axis system.

- If the problem describes a situation in which you know, from the start, what the object's direction of motion is, then orient one of your axes along the direction of motion.
  - In one dimension, this is simple since there is only one axis.
  - Some problems will tell you what the direction of motion is.
  - For some problems, the direction of motion is obvious (as in the case of an object sliding down a ramp).
- If you don't know what the direction of motion is, then orient your axes in such a way that one or more of the forces in the problem lie along an axis. The more forces you can do this with, the better.

b) Make clear what directions along each axis you are considering the positive and negative directions.

c) Represent the object as a point at the origin.

d) Draw each force acting on the object as an arrow (vector) with its tail at the origin pointing in the direction determined by problem's description of the physical situation. If the problem gives you information about a force's direction via an angle, indicate that on your diagram.

2) Find the total force along each axis direction

At this point, treat the problem as a vector addition problem (see "Vector Addition" handout).

- Get the x-component and y-component of each force (vector)
  - Notice that this is easy for the forces that are lined up along an axis
  - Make sure to give each components the proper sign
- Add like components
  - The sum of the x-components gives you $\bar{F}_{Tx}$ (the total force along the x direction).
  - The sum of the y-components gives you $\bar{F}_{Ty}$ (the total force along the y direction).

3) Apply Newton's Second Law ($\bar{F}_T = m\bar{a}$)

Now we apply Newton's Second Law along each or our axes.

We have

$$F_{Tx} = ma_x$$

and

$$F_{Ty} = ma_y$$

- In one dimension, we only have one equation
- If we knew the direction of motion of the object from the start and oriented our axis system so that one of our axes was along that motion, then we know the acceleration along the other axis is zero.

4) Getting the Acceleration

If the problem asks us to find the acceleration (magnitude and direction) of the object, we can use the equations above to get it. In this case, we have:

$$a_x = \frac{F_{Tx}}{m}$$

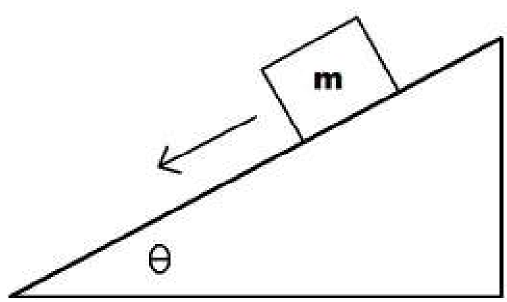
and

$$a_y = \frac{F_{Ty}}{m}$$

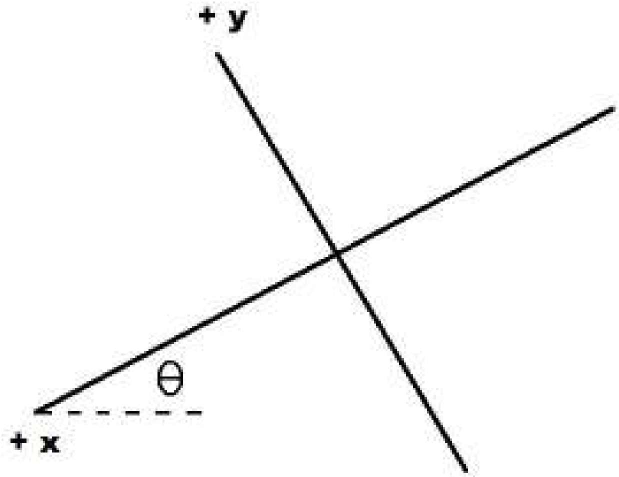
- In the case were we were able to orient one of our axes along the direction of motion, one of the acceleration components (a_x or a_y) will be zero. In this case, the sign in front of the non-zero acceleration will tell us the direction of the acceleration (relative to our axis system) and the absolute value will give us the magnitude of the acceleration.
- If we didn't know the direction of motion of the object from the start, then both a_x and a_y will probably be non-zero and we treat them as the components of a vector (because that's what they are). We get the magnitude and direction of the acceleration in the same way we get these for any vector (again, see "Vector Addition" handout).

Example 1

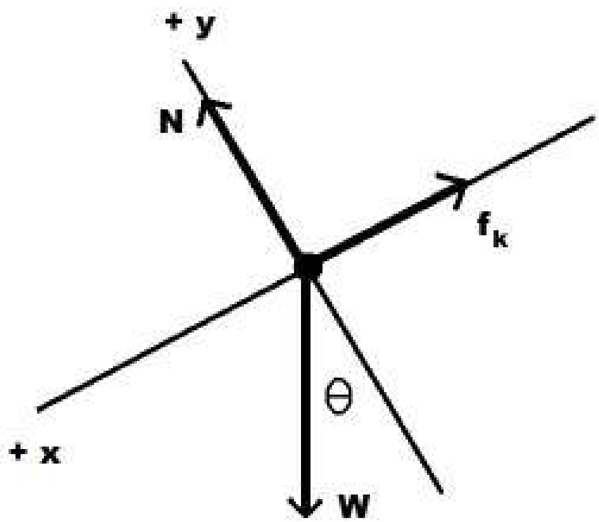
A block slides down a ramp with friction. The ramp makes an angle of 32° with the horizontal, the block has a mass of 3.5 kg and the coefficient of kinetic friction between the block and ramp is 0.41. Take the acceleration due to gravity to be 9.80 m/s^2. Determine the magnitude and direction of the block's acceleration.



1) Since I know the direction of motion of the block is down the ramp, I can set up my axis system so one of the axes (let's say the x-axis) is along the direction of the ramp.



I decided to make the positive x direction in the direction of motion and I've indicated the angle of the ramp since I might need this to determine the direction of one or more of the forces in the problem. Now let's draw in the forces. I'll represent the block as a dot at the origin and draw my forces off of it.



The normal force (N) is perpendicular to the contact surface, the kinetic friction ($f_k = \mu_k N$) is in the direction opposite to the motion and the weight (W = mg) is straight down and makes the same angle with the (negative) y axis as the ramp makes with the horizontal.

2) Now I treat this like a vector addition problem. First I need to get the x and y components of each of the three vectors in the above diagram:

$N_x = 0$ $\quad$ $f_{kx} = -\mu_k N$ $\quad$ $W_x = W \sin\theta$

$N_y = N$ $\quad$ $f_{ky} = 0$ $\quad$ $W_y = -W \cos\theta$

Now add like components to get the total force along each axis:

$$F_{Tx} = N_x + f_{kx} + W_x \quad \text{and} \quad F_{Ty} = N_y + f_{ky} + W_y$$

or

$$F_{Tx} = -\mu_k N + W \sin\theta \quad \text{and} \quad F_{Ty} = N - W \cos\theta$$

3) Making use of Newton's Second Law (and using W = mg) I get

$$-\mu_k N + mg \sin\theta = ma_x \quad \text{and} \quad N - mg \cos\theta = ma_y$$

or

$$-\mu_k N + mg \sin\theta = ma \quad \text{and} \quad N - mg \cos\theta = 0$$

In the last line, I used the fact that the motion is only along the x axis, so $a_x = a$ and $a_y = 0$.

4) I now have two equations with two unknown quantities ("a" and "N"). Using the y-component equations, I get

$$N = mg \cos\theta$$

If I plug this into the x-component equation, I get

$$-\mu_k mg \cos\theta + mg \sin\theta = ma$$

or

$$-\mu_k g \cos\theta + g \sin\theta = a$$

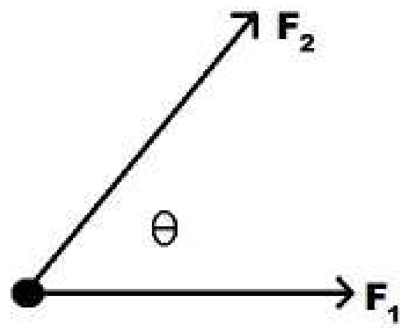
or

$$a = g(\sin\theta - \mu_k \cos\theta)$$

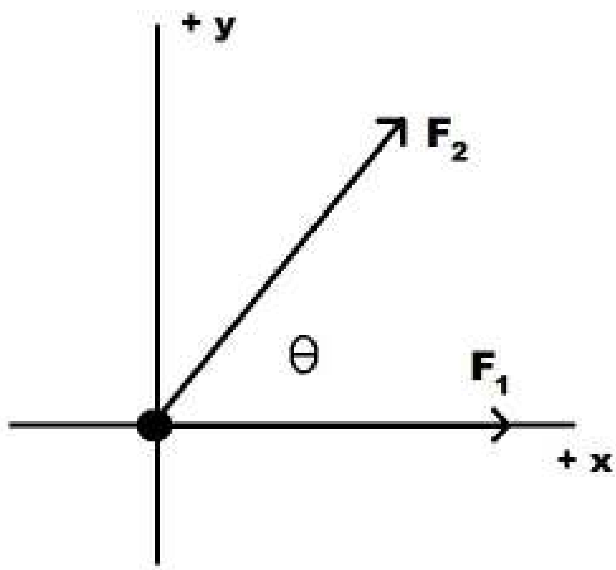
I can now plug in my values to get that: $a = 1.8\ m/s^2$. The acceleration is positive so that means the block is moving in the positive x direction (but we already knew that).

Example 2

An object of mass 2.2 kg is acted on by two forces, as indicated in the following diagram. The force magnitudes are F_1 = 4.5 N and F_2 = 5.4 N and the angle between the forces is 50°. Determine the magnitude and direction of the acceleration of the object.



1) In this case, I don't know the exact direction that the object will move in, so I'll set up my axis system so that at least one of the forces is lined up along an axis (F_1 in this case):



2) Now I add F_1 and F_2 as vectors. The x and y components are:

$F_{1x} = F_1$ $\qquad$ $F_{2x} = F_2 \cos\theta$

$F_{1y} = 0$ $\qquad$ $F_{2y} = F_2 \sin\theta$

Adding like components to get the total forces along the x and y directions gives

$$F_{Tx} = F_{1x} + F_{2x} \quad \text{and} \quad F_{Ty} = F_{1y} + F_{2y}$$

or

$$F_{Tx} = F_1 + F_2 \cos\theta \quad \text{and} \quad F_{Ty} = F_2 \sin\theta$$

3) Using Newton's Second Law, I get

$$ma_x = F_1 + F_2 \cos\theta \quad \text{and} \quad ma_y = F_2 \sin\theta$$

or

$$a_x = (F_1 + F_2 \cos\theta)/m \quad \text{and} \quad a_y = F_2 \sin\theta/m$$

Plugging in the given values, I get: $a_x = 3.6 \text{ m/s}^2$ and $a_y = 1.9 \text{ m/s}^2$

The x and y components of the acceleration are positive so I know the acceleration is up and to the right, relative to the origin (see diagram below).

4) The magnitude of the acceleration is given by: $a = \sqrt{a_x^2 + a_y^2}$ = 4.1 m/s^2

The direction is determined by: $\theta = tan^{-1}\left(\frac{a_y}{a_x}\right)$ = 28° where θ is the angle indicated below