# Basic Error Analysis

## Single Measurement Error

**Every measurement of a physical quantity (like a length or a duration of time) has an uncertainty (or "error") associated with it.**

**For example, if you measure the length of something with a meter stick which has millimeter markings, you might come up with a value**

$$X = 17.35 \text{ cm}$$

**In this case, the last digit – the "5" – is an estimate. In other words, the edge of the object seemed to fall half way between the 17.3 and 17.4 marks on the ruler.**

**However, since it's an estimate, you should give an indication of the degree to which you are unsure about that last digit. If you figure that the ruler is accurate (that the edges aren't worn down and that the ruler isn't warped) and that you are using it properly, you might suggest that**

$$X = 17.35 \pm .01 \text{ cm}$$

**But if the ruler is old and worn and you're being a little carless with the measurement process, it might be safer to record your reading as**

$$X = 17.35 \pm .1 \text{ cm}$$

**As a general rule, it's safe to use the "least count" of the measuring device as the uncertainty. The "least count" is the smallest unit for which the measuring device is set up to measure. For a meter stick with millimeter marking, the least count is 1 millimeter or 0.1 cm. For a digital voltmeter with a display that measured down to the hundredth volts (for example, 9.57 volts), the least count is 0.01 volts.**

**You should always indicate the error associated with each measured quantity. However, there is an indirect way of doing this. If I write the result of a measurement as**

$$X = 17.35 \text{ cm}$$

**I am implying that the uncertainty in this number is ± .01 cm. If I write is as**

$$X = 17.3 \text{ cm}$$

**I am implying that the uncertainty in this number is ± 0.1 cm.**

**<u>Significant Figures and Algebra</u>**

**You will notice that the last two values I wrote for X differed in the number of digits the quantity had – four for 17.35 cm and three for 17.3. We say that the first has four "significant figures" and the second had three "significant figures".**

**If, after measuring some quantities, we are required to plug them into a formula, it makes sense that the value we get from the formula should have a certain number of significant figures which is related to the number of significant figures of the numbers that where used to get it.**

**• Multiplication and Division**

**If we multiply and/or divide quantities, the value we get can't have more significant figures than the quantity with the least number of significant figures.**

**For example, let's say we have: x = 12.3, y = 2.3 and z = 3.456. If we plug these into the following formula**

$$f = \frac{x\,y}{z} = \frac{(12.3)(2.3)}{3.456}$$

**then, using our calculator, we get f = 8.185763889**

**But that's too many decimal places. In this case, the quantity with the least number of significant figures was y = 2.3, which had two significant figures. As a result, our final answer can't have more than two significant figures. Our final value, (after rounding up) is f = 8.2**

**• Adding and Subtracting**

**For addition and subtraction, the number of significant figures depends on the number of decimal places of the numbers being added and subtracted. The final answer can't be written out to more decimal places than the quantity with the least number of decimal places.**

**For example, let's say we have: x = 1.32, y = 5.344 and z = 3.4566. If we plug these into the following formula**

$$f = x + y - z = 1.32 + 5.344 - 3.4566$$

**then, using our calculator, we get f = 3.2074. Again, this is too many decimal places. Here, the quantity with the least number of decimal places is x = 1.32. So our final answer can't be written out to more than the second decimal place. Our final value (again, after rounding up) is f = 3.21**